On the variability of the LAI of homogeneous covers with respect to the surface size and application

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Abstract. The leaf area index (LAI) of different 100-m² forest plots was measured with accuracy at a resolution of 5 or 10 m². The results numerically confirm that the LAI can be considered as a weakly fluctuating surface parameter for homogeneous forests at sufficiently large scales. Furthermore, we show that this property of the LAI is a starting point for taking into account the surface heterogeneity in models involving LAI, when the scales are large enough. We have come up with a formula linking the PVI (perpendicular vegetation index) to the LAI, which allows one to calculate the LAI of mixed pixels. An accuracy indication of this formula is supplied.

1. Introduction

The leaf area index (LAI), which is the vertical projection of the foliage surface per unit of soil surface, is a key parameter for biomass evaluation. Despite the evidence of its natural definition and its importance in any model involving crop cover of the Earth, the LAI is not an easy parameter to measure (see, for example, Walter and Grégoire 1996). On a local scale, various methods exist but their application is strongly limited to scales of a few hundred or thousand square metres, which is out of proportion with the scales needed for global studies of the biomass. On these scales, satellite data are necessary and much work has been done to obtain the LAI from radiometric data. Asrar et al. (1984), Andrieu and Baret (1993), Price (1992), Goel and Qin (1994), for example, have entertained this question.

The main problem remains how to take into account the heterogeneity of the surfaces. In the present paper, we show, on the basis of three experiments described in §3, that the LAI can be considered, for a given homogeneous forest, or homogeneous textured forest, as a numerically stable parameter, if the soil’s area is...
sufficiently large. In §4 we show that the experiments lead to numerical estimations of this size. In §5, we point out a perpendicular vegetation index PVI-LAI model adapted to heterogeneous surfaces.

2. Notations and orientation

We shall first clarify the definition of the LAI adopted here. This is necessary, since different points of view exist. For example, it can be useful to consider surface foliage in terms of its vertical distance from the ground (Hapke 1993). Since we are interested in surface extension, we will use the classical definition of the LAI in direct relation to the soil’s surface.

If $\Omega$ is a horizontal soil surface of area $|\Omega|$, we consider the total area $S(\Omega)$ of all the leaves contained in the volume vertically projected on $\Omega$. The LAI is then defined at each point of the soil’s surface as the limit, when $d\omega=d|\Omega|\to 0$:

$$\text{LAI}(\omega) = \frac{dS}{d\omega}$$

(1)

This non-dimensional function is a strongly fluctuating function defined at each point $\omega$ of the soil’s surface. Fortunately, the total area of the leaf surface $S(\Omega)$ on large regions $\Omega$ is what particularly concerns us.

This leads to the mean LAI on the (large) scale $\Omega$:

$$\text{LAI}(\Omega) = \frac{S(\Omega)}{|\Omega|} = \frac{1}{|\Omega|} \int_{\Omega} \text{LAI}(\omega) d\omega$$

(2)

While (1) is a very fluctuating function of $\omega$, we may hope that its large-scale expression (2) is a stable function of $|\Omega|$. In other words, we may hope that for a given ‘homogeneous forest’ or ‘homogeneously textured forest’, defined below, when $|\Omega|\to +\infty$, we have a limit for the LAI

$$\frac{S(\Omega)}{|\Omega|} \to \lambda_V$$

(3)

where $\lambda_V$ is a constant depending on the type of vegetal cover under consideration.

We aim to show by experiment that, for homogeneous covers or textures of covers, the LAI can be considered as a constant surface parameter, when $|\Omega|$ is sufficiently large. Namely, we will show that for $\varepsilon$ chosen arbitrarily small, we can find a threshold size $|\Omega(\varepsilon)|$ such that

$$|\lambda_V - \frac{S(\Omega)}{|\Omega|}| < \varepsilon \text{ when } |\Omega| > |\Omega(\varepsilon)|$$

(4)

Thus, we may consider that the LAI is a constant surface parameter with a good approximation ($\varepsilon$) for $|\Omega|$ sufficiently large (larger than the threshold $|\Omega(\varepsilon)|$). One consequence of this property will be pointed out in §5.

2.1. A general definition of homogeneous textured forests adapted to LAI studies

Let us give a general definition for the homogeneity of forest LAI or the homogeneity of its texture. The notion of homogeneous forest is visually intuitive, as well as the one of statistical homogeneity of trees distribution. Nevertheless, the use of these notions for numerical results requires more quantitative, or at least accurate, definitions adapted to our purpose.
The classical notion of homogeneity in forestry depends on the distribution of the trunks, on the frequency, on the sociological statutes of the trees and species. This notion is adapted to forest management. For the present study, we need a concept adapted to the soil vertical projection of the leaf distribution.

Let us consider a surface $\Omega$ where trees are distributed in any number and manner. We observe the process of one leaf fall and its probability $P(\omega)$ to fall at the place $\omega$ of $\Omega$. We will say that the forest LAI (or simpler the forest) is homogeneous on $\Omega$, if $P(\omega)$ keeps the same constant value on $\Omega$. In other words, we suppose that after one leaf has fallen on the ground at $\omega$, the probability that the next leaf falls at any other position $\omega'$ of $\Omega$, has the same value. The position of $\omega$ can be considered as having a one-leaf size. Such a uniform distribution of the LAI corresponds to a very regularly distributed foliage (figure 1(a)).

Now, if the trees’ distribution and more generally foliage distribution is more sparse (figure 1(b)), the homogeneity concerns the statistical behaviour of the trees or foliage aggregates on $\Omega$. In this case, the same definition holds if we enlarge the size of $\omega$. Indeed, for $\omega$ sufficiently large, when the probability $P(\omega)$ to have the next fall of leaves within $\omega$ is the same as for any other $\omega' \in \Omega$ ($\omega'$ with the same size as $\omega$), the forest texture will be called homogeneous with respect to the LAI (figure 1(b)).

Figure 1(c) presents a forest texture that cannot be considered as homogeneous, at least in the field $\Omega$ of the figure. We observe that the first definition of a homogeneous (LAI) forest is a particular case of a homogeneous textured forest, when $\omega$ has the smallest possible size which is the leaf size.

Figure 1. Homogeneously textured forests in terms of horizontal LAI. The black regions represent the leaves' vertical projection on the ground surface. (a) A homogeneous forest, at the smallest scale of a leaf. (b) A homogeneous forest texture at the scale of $\omega$, larger than the leaf size. (c) A non homogeneous forest texture: one cannot find a grid size where the squares $\omega$ have the same probability to receive a leaf fall. Let us remark that it may become possible if we enlarge the observed area.
Remark. This definition of homogeneity is not easy to check in practice. The notion of homogeneity is in fact almost never directly accessible to measurements. Think for example of what is a homogeneous soil in geophysics, geology or geography. In most cases in natural sciences, when it has been defined with accuracy, the observation of homogeneity lies indirectly on imagery.

As a consequence of this definition of homogeneity, the accumulation of the leaves on the soil can be considered as a sum of a great number of processes having the same law. Thus, the classical Central-limit theorem allows us to consider that the soil’s leaf distribution is approximated by a normal law, in the sense that at \(\omega\), the probability \(Q\) of having \(n(\omega)\) leaves (or a leaf surface \(S(\omega)\)) follows the law \(dQ(n(\omega)) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}(n(\omega) - \mu)^2}\), where \(\sigma\) is the standard deviation of the distribution and \(\mu\), the mean number of leaves we want to estimate. The values of the mean and the standard deviations \textit{a priori} depend on the position \(\omega\) in \(\Omega\). Nevertheless, the homogeneity of the forest plot with the homogenization due to the wind effect, lead us to adopt the classical ergodicity hypothesis. This latter states that, if one considers all the potential realizations of the process in one point \(\omega\), the mean and standard deviation of the process are those of the unique observed realization of the process over the whole area \(\Omega\). In other words,

\[
\mu = \text{the average of measured leaves on } \Omega.
\]

Similarly, the standard deviation is that of the spatial distribution observed on \(\Omega\).

On the other hand, for a process following a normal law, we can compute the confidence interval for the measured mean value \(\mu\), as well as the standard deviation, even though this last value is not our main objective.

3. Description of the experiment

3.1. The areas of study

Three forest plots, A, B and C, were chosen for this study. In the French National Forests Office (ONF) their exact references are N° ONF 215, 003 and 1054. They are situated in the Alsacian plain (France) at 48°5’N, 7°55’E, with a mean altitude of 130 m. The forest present on plot A is basically formed by oak (\textit{Quercus robur} L.), beech (\textit{Fagus sylvatica} L.) and hornbeam (\textit{Carpinus betulus} L.). Plot B is composed of an almost pure beech grove. On plot C, the forest is an oak grove with beech undergrowth. The topography of the three plots is flat. Principal dendrometric features are given in table 1.

3.2. The sampling procedure

The estimation method for the leaf area index is derived from the ‘point quadrat’ method (Warren Wilson 1959, 1960, 1963) and applied to the leaf fall after the complete fall of leaves (Nizinsky and Saugier 1988, Sabatier 1989, Dufreène and Breda 1995). It consists of vertically introducing a thin needle of approximately 2 mm in diameter into the leaf fall and counting the pierced leaves stuck on the needle. This operation is repeated 2000 to 8000 times on the plot, to obtain an estimation of the leaf fall. In doing this counting operation, one must take into account only the leaves of the current year. This is generally not very difficult, since the decomposition of the leaves of the previous years is easily recognisable.

The mean number of leaves, obtained by repeating this operation, is a reliable estimation of the value \(S(\Omega)\). The variation of the number of leaves from place to place is proportional to the variations of \(S(\Omega)\).
Table 1. Dendrometric characteristics of experimental plots.

<table>
<thead>
<tr>
<th>Plot N°</th>
<th>Density (trees/ha$^{-1}$)</th>
<th>Relative density</th>
<th>Dbh diameter at breast height</th>
<th>Age (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (N° ONF 215)</td>
<td>269</td>
<td>66</td>
<td>34</td>
<td>70–80</td>
</tr>
<tr>
<td>beech</td>
<td>176</td>
<td>89</td>
<td>23.8</td>
<td>50–60</td>
</tr>
<tr>
<td>oak</td>
<td>49</td>
<td>52</td>
<td>125</td>
<td></td>
</tr>
<tr>
<td>hornbeam</td>
<td>44</td>
<td>22.5</td>
<td>30–40</td>
<td></td>
</tr>
<tr>
<td>B (N° ONF 1054)</td>
<td>381</td>
<td>89</td>
<td>23.8</td>
<td>50–60</td>
</tr>
<tr>
<td>beech</td>
<td>339</td>
<td>11</td>
<td>40.5</td>
<td>115</td>
</tr>
<tr>
<td>oak</td>
<td>42</td>
<td>40.5</td>
<td>115</td>
<td></td>
</tr>
<tr>
<td>C (N° ONF 3)</td>
<td>1039</td>
<td>85</td>
<td>11.5</td>
<td>25–30</td>
</tr>
<tr>
<td>beech</td>
<td>881</td>
<td>15</td>
<td>50</td>
<td>123</td>
</tr>
<tr>
<td>oak</td>
<td>158</td>
<td>50</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

By this technique, a sampling has been made of the three 100 m × 100 m plots A, B and C (figure 2(a)). It was carried out in November 1999, at the tail end of autumn when the leaves were all down. Specifically, plot A was divided into a 5 m × 5 m grid (figure 2(a)), whereas a 10 m × 10 m grid was used for plots B and C. Within each elementary square, the counting operation with the needle was renewed 20 times (figure 2(b)). Then, the mean of the 20 samples was computed and associated to the elementary square. In the following, we will consider the asymptotic value of these numbers when the surface of the aggregation of the elementary squares increases. The increasing surfaces Ω$_i$ from one elementary square to the total one Ω$_{tot}$, is presented in figure 2(c).

Figure 3(a) represents in grey levels the values of the mean number of contacts per elementary square in plot A. The densest S(Ω$_i$) corresponds to the clearer grey.

Figure 2. Scheme of the sampling procedure. (a) A plot of forest and the elementary squares. (b) One elementary square in which 20 measurements are made. (c) The increasing surfaces are built as the aggregation of the elementary squares Ω$_i$ ($i=1$ to 400, for plot A and $i=1$ to 100 for plots B and C) following the dashed arrow. These aggregations constitute squares of increasing size $X$. One has $X=5$ m to 100 m for plot A and $X=10$ m to 100 m for plots B and C.
Figure 3. The leaf surface measurements $S(\Omega_i)$ for $\Omega_i \in \Omega_{tot}$ in grey levels. The higher density corresponds to the white grey level. (a) Plot of forest A. In grey level, the representation of the mean of the 20 leaf surface measurements on each elementary squares. (b) Plot of forest A. Contour plot of the previous data. The circles approximately represent the texture elements. They appear in a more quantitative way in figure 4. (c) and (d) The same legend as (b) for the plots of forest B and C. The radius of the superimposed circles have the values of the thresholds as shown in figure 5.

level. Figure 3(b), (c) and (d) represents the contour lines of the values $S(\Omega_i)$ for plots A, B and C, with the same grey level convention. We may perceive the presence of trees.

3.3. Properties of the data

3.3.1. Statistical properties

Let us first look at the statistical properties of the previous data, before looking at the objective, which is to evaluate the variations of the foliage area, $S(\Omega)$ when $|\Omega|$ increases. The results are presented table 2 and figure 4. This figure shows the Gaussian behaviour of the data. We remark furthermore that these values are spatial means on one given realization of the process. Then, the homogeneity of the forest plots A, B and C and the ergodicity property, mentioned at the end of §2, are indirectly confirmed. More precisely, these observations must be satisfied if the forest is homogeneous and the ergodicity property is justified, but they are not sufficient to assert that the homogeneity and ergodicity hold.

3.3.2. The relation between the number of contacts and the surface foliage

The paper focuses on the asymptotic behaviour of $S(\Omega)/|\Omega|$ when $|\Omega|$ increases from one elementary square to the total one $\Omega_{tot}$. The number of leaves estimated in each elementary square from the 20 measurements is proportional to the value $S(\Omega)$. This property is sufficient for the asymptotic study. Nevertheless, this mean
Table 2. Statistical properties of the data. The confidence interval centred at $\lambda_{\text{tot}}$ contains the asymptotic value $\lambda_e$ with a probability of 0.95. Its half-length contains the maximum of the difference $|\lambda_e - \lambda_{\text{tot}}|$. We have $\lambda_{\text{tot}} = 6.15$ for plot A, $\lambda_{\text{tot}} = 5.05$ for plot B, $\lambda_{\text{tot}} = 5.35$ for plot C.

<table>
<thead>
<tr>
<th></th>
<th>Plot A</th>
<th>Plot B</th>
<th>Plot C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample size</td>
<td>400</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Mean</td>
<td>6.15</td>
<td>5.05</td>
<td>5.35</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.31</td>
<td>0.70</td>
<td>1.15</td>
</tr>
<tr>
<td>Median</td>
<td>6.05</td>
<td>5.05</td>
<td>5.20</td>
</tr>
<tr>
<td>Mode</td>
<td>6.00</td>
<td>5.05</td>
<td>4.55</td>
</tr>
<tr>
<td>Min</td>
<td>1.50</td>
<td>3.60</td>
<td>3.15</td>
</tr>
<tr>
<td>Max</td>
<td>12.20</td>
<td>7.45</td>
<td>8.70</td>
</tr>
<tr>
<td>Confidence interval</td>
<td>[6.02, 6.28]</td>
<td>[4.91, 5.18]</td>
<td>[5.12, 5.58]</td>
</tr>
</tbody>
</table>

$\nu = \frac{\text{Max}|\lambda_e - \lambda_{\text{tot}}|}{\lambda_{\text{tot}}}$ (at the level of 95%)

2.09% 2.75% 4.26%

Figure 4. Histograms of the leaves’ number measurements. We observe the gaussian behaviour of the curves.

value is in fact a direct approximation of $S(\Omega)$, regardless of the leaf size. Let us first consider that the leaves have a large surface. Now, suppose that without moving them, they are cut at the size of small leaves. The number of contacts obtained with the needles will keep in the mean, the same value for $S(\Omega)$. This value is then independent of the leaf size and represents the mean surface foliage $S(\Omega)$. The same argument holds for small leaves when, without moving them, we suppose that they are aggregated in large surfaces.

3.3.3. A general remark on the asymptotic value of a set of surface measurements

Without any information on the measured surface foliage, we may have numerical results on the asymptotic behaviour of the value $S(\Omega)/|\Omega|$, provided that the measured values obtained with the needles stay bounded. The general result is proved in the appendix. It shows that, regardless of the meaning of the measured $S$, provided that $S \in [0, 7]$, we must have a size $\sqrt{|\Omega|} \geq 93\text{m}$ for $\nu = 1$, in (4). We shall see that this general result will be improved with the actual leaf surface measurements. This
improvement is due to the fact that the LAI distribution, even though fluctuating at high resolution, is not arbitrarily erratic as considered in the appendix.

4. The results

Figure 5 is a representation of the experimental values $S(\Omega)/||\Omega||$ as measured on the three plots, with respect to increasing areas $||\Omega||$. The surface foliage is counted following the dotted line path represented in figure 1(c), elementary square by elementary square. For each plot, we have effectuated this computation starting from each of the four corners of the plot. In fact, we have four realizations of the same process. They are not independent processes, since they have the same mean, but we can consider that for the study of the spatial variability, they are independent.

We observe three different segments on these curves. The first one, between the origin and the first vertical line, is strongly fluctuating. The second one is more stable but presents notable differences with the mean. Finally, the third region is weakly fluctuating. In figure 5(a) the first segment is approximately limited by the abscissa value 5 m (top of figure 5(a)). This corresponds to a square of approximately $5^2 \text{ m}^2$. This is the approximate size of the foliage aggregates within the trees. The first segment of plots B and C respectively are 17 m$^2$ and 28 m$^2$. On parcel B, as on A, this is the size at which the foliage and holes reveal a texture. The case of plot C is less clear, due to the heterogeneity of the parcel, as can be observed in figure 3(c). Nevertheless, figure 5(c) shows that a sort of texture appears beyond the scale of 28 m.

The second segment appearing on these curves corresponds to the type of uniformity of the foliage and hole textures.

Beyond these scales, from 35 m on parcel A, 48 m on B and 50 m on C, the tree distribution interferes with the smoothing effect of the averaging of the data. The total effect is that the fluctuations are smoother. Let it be noted that on the third part of the curves we must observe separately the four samples, because, by construction, their values are not strictly independent up to 50 m. Nevertheless, the smoothness of each curve remains significant. We observe that they differ very little from the mean $\lambda_{\text{tot}}$ of the LAI of the total plot.

A visual appreciation of these sizes in figure 3 confirms the values. They are pointed out in this figure as circle diameters. Let us remark that the computation of the variogram, which would give a numerical value for the evidence of the textures, is not significant here, because the quantity of data is too weak.

One of the consequences of the discussion of §2 is that the computation of the confidence intervals for a normal distribution is allowable. Table 2 contains the confidence intervals, which give bounds within which the value $\lambda_V$ of the LAI is expected to lie with a probability of 95%. We recall that we note $\lambda_{\text{tot}}$ the actual measured LAI of the whole plot $\Omega_{\text{tot}}$, while $\lambda_V$ is the constant LAI expected for an ‘infinite’ forest having the same texture as the one observed in each experimental plot.

Thus, for such a forest, with the same texture as plot A, if one takes the constant value $\lambda_V = 6.15$ for the LAI, which is the measured value $\lambda_{\text{tot}}$ for the LAI of the plot, with a probability of 0.95, error is less than 2.09% (table 2). Similarly, if one takes $\lambda_V = 5.05$ in a forest of the type of plot B, the error is less than 2.75% with the same probability. To take the constant LAI $\lambda_V = 5.34$ for a forest having the same texture of plot C leads to a greater error, 4.26%. This percentage, larger than the previous one, is obviously due to the fact that, for this type of texture (see figure 3(d)), the size of 100 m is not large enough for a better accuracy.

Let us return to the question of the size $\Omega(\varepsilon)$ for a given precision $\varepsilon$ on the LAI,
Figure 5. (a) Plot of forest A. The lower abscissa is a counting of the elementary squares $\Omega_i$ taken into account along the dashed arrow of figure 1(c). The upper abscissa is the size $X$, in metres, of the increasing squares of figure 1(c). The ordinate represents the LAI. We plot four curves corresponding to the same experiment repeated when we start from the four corners of parcel A. The vertical lines represent the apparent thresholds on the textures, as they could be observed on figure 3. (b) and (c) Same legend for plots of forests B and C.
as posed in (4). If we note $\zeta$, for each plot, the maximum difference $|\lambda_V - \lambda_{tot}|/\lambda_{tot}$ (table 2), we have

$$|\lambda_V - \lambda_{tot}| \leq \varepsilon \lambda_{tot}$$  \hfill (5)

Then from the curves of figure 5, we deduce from the experiment, an upper bound for the value

$$\left| \lambda_{tot} - \frac{S(\Omega)}{\Omega} \right|$$  \hfill (6)

taking for each plot the maximum of the difference between the four curves and their mean value $\lambda_{tot}$. Let us note $\Phi(\Omega)$ this upper bound. Combining (5) and (6), we have

$$\left| \lambda_V - \frac{S(\Omega)}{\Omega} \right| \leq \varepsilon \lambda_{tot} + \Phi(\Omega) = \varepsilon$$  \hfill (7)

The maximum of the right-hand side of (7) is presented in figure 6 for each plot, with respect to the area $|\Omega|$ of $\Omega$. It is approximated by a smooth bold curve.

In the right-hand side of (7), the term $\varepsilon \lambda_{tot}$ defined by the confidence interval cannot tend to zero when the area increases. This term depends on how well the statistics represent the given experiment. It should tend to zero if the size of the plot would be larger.

We recognize that the more the forest is homogeneously textured, the more these curves decrease. To use them, we chose a value for $\varepsilon$ (expressed in LAI units) and obtain an approximation of the minimum size of the square for which the LAI will be approximated with an error less than $\varepsilon$. For example, for a relative error on the LAI less than 1 (i.e. 17%), figure 6(a) shows that we need approximately a square region $\Omega(\varepsilon)$ of 75 m $\times$ 75 m at least. If we come back to the rough approximation given in the last paragraph of §3.3, we note a better result for $\Omega(\varepsilon)$, due to more precise considerations on the data than the general arguments on the surfaces.

We obtain (figure 6(b)) a size of 20 m for $\varepsilon = 1$ (i.e. a maximum error of 20% on $\lambda_V$) and 50 m for $\varepsilon = 0.82$, on plot B. For plot C, the same square size of 50 m leads to a relative error of 32% on the LAI (figure 6(c)). For this type of texture the area has to be larger to allow a constant value $\lambda_V$ for the LAI.

5. Application: A LAI-PVI formula for remote sensing

Some applications of these results can now be considered. A first question arises: is it possible to make catalogues of constants $\lambda_V$ for given regions, from a local study and use these catalogues for remote sensing? The present paper does not give the elements for a well-argued answer. Nevertheless the question could be answered for the summer season LAI, if one considers a more consequent number of test sites and textures. Computer simulations are a possibility. We shall now turn our attention to another problem.

5.1. The LAI of mixed pixels from visible and near-infrared channels

We consider a pixel $\Omega$ composed of forest with a homogeneous texture and various bare soils. This pixel is observed by a radiometer in the visible and near-infrared channels. We shall give a relation between the LAI of $\Omega$ and the visible and near-infrared reflectivities. The role of the hypothesis made below in the accuracy of the result, will be discussed in §5.2.
Figure 6. Relation between the size $\sqrt{\Omega(e)}$ of the ground square and the distance $e$ of the asymptotic LAI, $l_v$, to the LAI of $\Omega(e)$. These (bold) curves are the right hand side of (7). They are the maximum of the expression (12), taking into account the four realizations of $S(\Omega)/\Omega$ presented in figure 4. The bold curves are fitted lines of hyperbolic form. The left ordinates are the value of $e$ in LAI units. The right ordinates are the previous values in percent of the total measured LAI ($l_{tot} = 6.15$ for plot A, $l_{tot} = 5.05$ for plot B, $l_{tot} = 5.35$ for plot C).

We suppose first that the LAI of a forest region is equal to $l_v$ and that this constant is known, for example after a local terrain experiment. We suppose also that the region is composed of a mixture of this forest and various bare soils, with
different degrees of wetness (figure 7(a)). We suppose furthermore that the soil line is known (figure 7(b)). We suppose finally that the forest is radiometrically homogeneous, in other words, that we have at any point of the forest (bold dashed region, figure 7(a)) the same visible and near-infrared reflectivities \( L_{\text{vis}} \) and \( L_{\text{nir}} \) which constitute the vector \( \mathbf{L}_v = (L_{\text{vis}}, L_{\text{nir}}) \).

The PVI (Richardson and Wiegand 1977) of a mixed soil-pixel \( \Omega \) having the radiance \( \mathbf{L}(\Omega) = (L_{\text{vis}}(\Omega), L_{\text{nir}}(\Omega)) \) is defined as the distance of \( \mathbf{L}(\Omega) \) to the soil line (figure 7(b)). In other words, its explicit value is given by

\[
PVI(\mathbf{L}(\Omega)) = \frac{|L_{\text{nir}}(\Omega) - (aL_{\text{vis}}(\Omega) + b)|}{\sqrt{a^2 + 1}}
\]

where \( a \) and \( b \) are defined by the soil line equation (figure 7(b)). We will denote \( \Omega_v \)

![Figure 7](image.png)

Figure 7. (a) The type of region we may consider for the application of the LAI/PVI formula (15). Each of the 25 squares constitutes a mixed pixel noted \( \Omega \) in the text. Its radiance is \( \mathbf{L}(\Omega) \). (b): The radiometric domain corresponding to the type of region (a), where the various surface components are radiometrically homogeneous. The length of the segment which joins \( \mathbf{L}(\Omega) \) to \( \mathbf{L}(\Omega_v) \) is \( \frac{\Omega_v}{|\Omega|} \), while the length of the segment which joins \( \mathbf{L}(\Omega) \) to \( \mathbf{L}_v \) is equal to \( \frac{\Omega_v}{|\Omega|} = 1 - \frac{\Omega_v}{|\Omega|} \).
the forest part of $\Omega$ and $\Omega_s$ its soil component, which is a mixture of the various types of soil (note that in this section, $\Omega_r$ plays the role of $\Omega$ in the previous section). In this ideal case, the PVI of the mixed pixel $\Omega$ has exactly the value $|\Omega_r|/|\Omega| \text{PVI}(L_r)$. Indeed (figure 7(b)), the radiance of the mixed pixel is

$$L(\Omega) = \frac{|\Omega_r|}{|\Omega|} L_r + \frac{|\Omega_s|}{|\Omega|} L(\Omega_s) = \frac{|\Omega_r|}{|\Omega|} L_r + \left(1 - \frac{|\Omega_r|}{|\Omega|}\right) L(\Omega_s)$$

where $\Omega_s$ is the part of $\Omega$ containing all the soils (i.e. the complementary part of $\Omega_r$). Thus,

$$L(\bar{\Omega}) - L(\bar{\Omega}_s) = \frac{|\Omega_r|}{|\Omega|} (L_r - L(\Omega_s))$$

A glance at the triangles of figure 7(b) shows that this equality is equivalent to

$$\text{PVI}(L(\bar{\Omega})) = \frac{|\Omega_r|}{|\Omega|} \text{PVI}(L_r)$$

(8)

On the other hand, the actual LAI of the mixed pixel $\bar{\Omega}$ is given by

$$\text{LAI}(\bar{\Omega}) = \frac{S(\bar{\Omega})}{|\Omega|} = \frac{S(\Omega_s)}{|\Omega|} = \lambda_r \frac{|\Omega_r|}{|\Omega|}$$

(9)

where, as previously, $S(\bar{\Omega})$ is the total area of the foliage vertically projected onto the mixed region $\bar{\Omega}$.

Then, in the ideal case we consider, we deduce, from (8) and (9), that the LAI of the mixed pixel $\bar{\Omega}$ satisfies

$$\text{LAI}(\bar{\Omega}) = \frac{\lambda_r}{\text{PVI}(L(\bar{\Omega}))} \text{PVI}(L_r)$$

(10)

This is an exact formula if the hypotheses are exactly satisfied.

We may verify that when $\bar{\Omega}$ is completely covered by forest, we have $L(\bar{\Omega}) = L_r$, then (10) gives LAI($\bar{\Omega}$) = $\lambda_r$. When the pixel is uniquely covered by bare soils, $L(\bar{\Omega})$ belongs to the soil line, then, PVI($L(\bar{\Omega})$) = $\lambda_r = 0$ and LAI($\bar{\Omega}$) = 0. In all the other cases, we have $\text{LAI}(\bar{\Omega}) \in [0, \lambda_r]$.

5.2. Error estimation of formula (10)

Let us now consider how the various approximations induce errors regarding the LAI computed by (10). The approximations are twofold. They concern the radiometric part of (10) and the value $\lambda_r$.

The first hypothesis is that the vegetation is homogeneous, i.e. that its radiometric domain represented in figure 7(b) by a dashed area is exactly reduced to a known point $L_r$. The same argument holds for the soils, which are aligned in a known manner. Then, instead of $|\Omega_r|/|\Omega| = \text{PVI}(L(\bar{\Omega}))/\text{PVI}(L_r)$ coming from (8), we have

$$\frac{|\Omega_r|}{|\Omega|} = \frac{\text{PVI}(L(\bar{\Omega}))}{\text{PVI}(L_r)} + \varepsilon_{\text{PVI}}$$

(11)

where $\varepsilon_{\text{PVI}}$ is an unknown value, which can nevertheless be estimated, as we shall see later.

The second source of error comes from the approximation discussed in this paper. Namely, the exact mean value $\lambda_r$ of the LAI of a forest with a uniform texture...
is not exactly known because the region $\Omega_r$ is not infinite (term $\Phi$ in (7)) and the texture is not absolutely uniform (term $\lambda_{tot}$). The first two equalities of (9) are always true. The third equality means that

$$\frac{S(\Omega_r)}{|\Omega_r|} = \lambda_V$$

while, coming back to (9), we have only

$$\frac{S(\Omega_r)}{|\Omega_r|} = \lambda_V + \epsilon_{\lambda_V}$$

where $S(\Omega_r)/|\Omega_r|$ is the estimated value of the LAI on a limited surface $\Omega_V$, $\lambda_V$ the theoretical asymptotic value of the LAI and where the error $\epsilon_{\lambda_V}$ satisfies

$$\epsilon_{\lambda_V} \leq 2\lambda_{tot} + \Phi(\Omega_r)$$

Finally, if we come back to (10), the actual terrain value of the LAI of the mixed pixel $\Omega$ is

$$\text{LAI}_{\text{actual}}(\Omega) = \left( \frac{\text{PVI}(L(\Omega))}{\text{PVI}(L_r)} \right) \text{LAI}_{\text{computed}}(\Omega) + \epsilon_{\text{PVI}} \lambda_V$$

At the first order, i.e. if we neglect the term $\epsilon_{\lambda_V}$, the ideal expression (10) is then replaced by

$$\text{LAI}_{\text{actual}}(\Omega) = \lambda_V \text{PVI}(L(\Omega)) + \left[ \epsilon_{\beta} \text{PVI}(L(\Omega)) + \epsilon_{\text{PVI}} \lambda_V \right]$$

where the term in the brackets is the gap between the actual LAI of the pixel and the crude application of the formula (10) with the various necessary approximations detailed behind. Namely, the term $\lambda_V \text{PVI}(L(\Omega))/\text{PVI}(L_r)$ in (13) is the computed LAI with an approximated soil line and a choice of $L_r$. Then, (13) can be summarized by

$$\text{LAI}_{\text{actual}}(\Omega) = \text{LAI}_{\text{computed}}(\Omega) + [\text{error}]$$

Let us estimate each term in the brackets of equation (13). From (11) and (12), at the first order, we have

$$\epsilon_{\beta} \frac{\text{PVI}(L(\Omega))}{\text{PVI}(L_r)} \leq (2\lambda_{tot} + \Phi(\Omega_r)) \frac{|\Omega_r|}{|\Omega|}$$

We have already studied the first factor $(2\lambda_{tot} + \Phi(\Omega_r))$ and represented its values (figure 6), for three types of forest texture. Figure 8 shows the values of the right-hand side of (14) when the vegetation content $|\Omega_r|/|\Omega|$ increases from 0 to 100%.

The term $\epsilon_{\text{PVI}}$ can also be estimated with respect to the radiometric heterogeneity of the vegetation and the soil’s components of $\Omega$. The general exact value of $\epsilon_{\text{PVI}}$ can be obtained from Raffy (1994), but the particular expression of the PVI (of linear type) leads to an approximate value of $\epsilon_{\text{PVI}}$ useful for the rough estimate we need in the present study of (10). Figure 9(a) gives an estimation of this value as the length of the bold arrow. This length is obviously defined by the size of the radiometric heterogeneity of soils and vegetation.

Let us remark that we must consider that the radiometric extensions of the
Figure 8. The upper bound of the error due to the hypothesis on the radiometry. For example, if the percentage of the vegetation component is $V/\bar{V} = 50\%$, the error on the LAI will be majored by 0.5 for plot C, while it is majored by 0.3 for plot A. The curves are simply obtained by the multiplication of the values presented in figure 5 in bold lines, by the vegetation percentage.

Dashed regions (figure 9(b)) are due to pure elements of soils and vegetation within $\Omega$. If we take ‘Soils’ as the size of the soil’s radiometric domain, ‘Veg’ as the vegetation and ‘Mixed’ as the complementary part of the two previous, figure 9(c) shows that

$$e_{PVI} = \frac{1}{2(1 + \text{Mixed/Soils})} + BL \frac{\text{Veg} - \text{Soils}}{2(\text{Mixed} + \text{Soils})(\text{Mixed} + \text{Veg})}$$

(15)

where $BL$ is the length of the segment $BL$ (figure 9(c)). Soils, Veg and Mixed represent lengths in radiances or reflectivities units. The numerical value of (15) is not unit dependent. In comparison with the classical radiometric domains for usual forest and soil landscapes, we know that the factor of $BL$ can often be small in comparison with the first term of (15). Thus an approximation of (15) is

$$e_{PVI} \approx 1/2(1 + \text{Mixed/Soils})$$

(16)

This expression tends to zero when the Soils extension tends to zero and takes the value 0.25 when the radiometric extension of Mixed surfaces is comparable to that of the soils. Notice that the radiometric heterogeneity of the vegetation does not appear directly in the approximation (16), but only through the ‘Mixed’.

Finally, if we imagine a large radiometric heterogeneity for the soils, such that $\text{Mixed/Soils} = 1$ and if we suppose that the forest percentage in $\Omega$ is 50%, a rough idea of the error into brackets in (13) is given by

$$0.25 + \lambda_x \cdot 0.25 \approx 1.75$$

(expressed in LAI unit)

for a textured forest as plot A with a LAI of 6, while for a texture comparable to that of plot C with a LAI of 3.5, this error becomes roughly

$$0.5 + \lambda_x \cdot 0.25 \approx 1.37$$

(17)

Table 3 presents some values of this error in percentage of LAI, for different types of radiometric and texture heterogeneity. The error increases for weakly textured forests because the size of our plot was not large enough for a good accuracy.
Figure 9. (a) For the radiance $\mathbf{L}(\Omega)$ of the mixed pixel, the percentage of vegetation is the size of the arrow in bold. (b) When the soils radiances are no more along a line, the extreme values of the vegetation percentage presented in (a) are defined by the double arrow. Since $\varepsilon_{\text{PVI}}$ is the maximum difference between the latter percentage and the extreme values, the length of the double arrow is twice $\varepsilon_{\text{PVI}}$. (c) The length of segment AB is the maximum radiometric extension of the soils along the line $\mathbf{AL}$. It is noted ‘soils’. Similarly, the length of segment DC is noted ‘Veg’. The segment ‘Mixed’ is the segment contained in the two previous ones.
Variability of LAI and its application

Table 3. Maximum values of the relative errors on the LAI obtained from the expression in the right-hand side of (13). The values are expressed following two different values of the LAI constant measured on the given plot of forest A, B and C, $\lambda_\nu=6$ and 3. The errors take into account the weak size of the experimental plots, since the confidence interval is included. The maximum errors are given for various radiometric heterogeneities.

<table>
<thead>
<tr>
<th>Radiometric heterogeneity</th>
<th>Mixed/Soils (figure 9) = 1</th>
<th>Mixed/Soils = 5</th>
<th>Mixed/Soils = 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plot of type A</td>
<td>29%</td>
<td>12.5%</td>
<td>8.6%</td>
</tr>
<tr>
<td>Plot of type C</td>
<td>33.3%</td>
<td>16.6%</td>
<td>12.8%</td>
</tr>
<tr>
<td>$\lambda_\nu=3$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plot of type A</td>
<td>33.3%</td>
<td>16.6%</td>
<td>12.8%</td>
</tr>
<tr>
<td>Plot of type C</td>
<td>41.6%</td>
<td>25%</td>
<td>21%</td>
</tr>
</tbody>
</table>

on $\lambda_\nu$. That gives indications for possible further experiments for catalogues of $\lambda_\nu$, as suggested at the beginning of the section. A better precision on $\lambda_\nu$ would reduce the term 0.5 in (17) which plays a part of $0.5/\lambda_\nu=16.5\%$ for $\lambda_\nu=3$ in the values of table 3.

6. Conclusion

The spatial variability of the forest’s LAI, as a surface parameter, in opposition to its vertical variability, has been approached in a quantitative way. Depending on the type of texture encountered, we obtain various results, which present common aspects.

Despite the very high variability on the scale of a few metres, we observed a decreasing variability with respect to the size of the soil’s surface.

This allows two perspectives, at least. First, the study of the significance of LAI catalogues, in specific regions of the Earth. This question has only been posed by the present paper.

The second perspective is an application more adapted to remote sensing. It is a relation between the PVI and the LAI of mixed pixels. The study of the error due to the application of the formula to real situations is presented and evaluated. This is an invitation for other more systematic terrain experiments of the type presented here.

Appendix

Let us consider (figure A(a)), a positive surface parameter $S$ defined for $\omega \in \Omega_{\text{tot}}$, and bounded by $\max_{\omega \in \Omega_{\text{tot}}}(S(\omega))$, regardless of its physical significance. Following the procedure, defined in figure 1, for the aggregation of the elementary squares $\Omega = \Omega_1$, $\Omega = \Omega_1 \cup \Omega_2$, ..., $\Omega = \Omega_1 \cup \Omega_2 \cup \ldots \cup \Omega_n$, ..., we obtain a sequence of the mean values of $S(\Omega)$. This sequence has an asymptotic behaviour. In the appendix, we show this general property.

For an intermediate surface $\Omega = \bigcup_{i=1}^{N^2} \Omega_i$ (figure A(b) and (c)), we can easily observe
that

\[
\left| \frac{1}{|\Omega|} \sum_{i=1}^{400} S(\Omega_i) - \frac{1}{|\Omega|} \sum_{i=1}^{N^2} S(\Omega_i) \right| \leq \text{Max}(S(\Omega_i)) \left( 1 - \frac{N^2}{400} \right) \tag{A1}
\]

where \( N^2 \) is the number of elementary squares in the intermediate \( \Omega \) and \( 400 = 20^2 \) is the number of elementary squares in \( \Omega_{\text{tot}} \). Now, if we consider that in \( \Omega_{\text{tot}} \), we reach the value \( \lambda_{\text{tot}} = 1/|\Omega_{\text{tot}}| \sum_{i=1}^{400} S(\Omega_i) \) and take into account the measurements which show that \( \text{Max}(S(\Omega_i))_{i=1,400} \leq 7 \) (expressed in leaf surface in the paper), one deduces from (A1), that

\[
\left| \lambda_{\text{tot}} - \frac{S(\Omega)}{|\Omega|} \right| \leq 7 \cdot \left( 1 - \frac{N^2}{400^2} \right)
\]

where \( \Omega \) is an intermediate surface between the elementary 5 m x 5 m surface and the total \( \Omega_{\text{tot}} \). More generally, this inequality can obviously be written

\[
\left| \lambda_{\text{tot}} - \frac{S(\Omega)}{|\Omega|} \right| \leq \text{Max}(S(\omega)) \left( 1 - \frac{|\Omega|}{|\Omega_{\text{tot}}|} \right)
\]

Compared with (4), this inequality shows that \( [\lambda_{\text{tot}} - (S(\Omega)/|\Omega|)] \leq \varepsilon \) as soon as \( |\Omega| \geq |\Omega_{\text{tot}}|(1 - \varepsilon/\text{Max}(S(\omega))_{\omega \in \Omega}) \). This result, obtained by simple geometrical
considerations, is quite rough. Indeed, with the numerical values of plot A, we obtain

$$\lambda_{\text{tot}} - \frac{S(\Omega)}{|\Omega|} \leq 1 \text{ (in LAI unit), for } |\Omega| \geq 0.86|\Omega_{\text{tot}}|$$

Then, the size of the square $\Omega$ must satisfy

$$\sqrt{|\Omega|} \geq \sqrt{0.86|\Omega_{\text{tot}}|} \approx 93 \text{ m}$$

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